

Paper Reference(s)

**6666/01**

# **Edexcel GCE**

## **Core Mathematics C4**

### **Silver Level S1**

**Time: 1 hour 30 minutes**

**Materials required for examination**  
**papers**

Mathematical Formulae (Green)

**Items included with question**

Nil

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

#### **Instructions to Candidates**

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Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

#### **Information for Candidates**

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A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 8 questions in this question paper. The total mark for this paper is 75.

#### **Advice to Candidates**

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You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

#### **Suggested grade boundaries for this paper:**

<b>A*</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<b>67</b>	<b>59</b>	<b>53</b>	<b>47</b>	<b>40</b>	<b>34</b>

1. 
$$f(x) = \frac{1}{\sqrt[3]{4+x}}, \quad |x| < 4.$$

Find the binomial expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^3$ . Give each coefficient as a simplified fraction.

(6)

June 2009

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2. (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \quad |x| < \frac{8}{3},$$

in ascending powers of  $x$ , up to and including the term in  $x^3$ , giving each term as a simplified fraction.

(5)

- (b) Use your expansion, with a suitable value of  $x$ , to obtain an approximation to  $\sqrt[3]{7.7}$ . Give your answer to 7 decimal places.

(2)

January 2008

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3. 
$$f(x) = \frac{6}{\sqrt[3]{9-4x}}, \quad |x| < \frac{9}{4}.$$

- (a) Find the binomial expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^3$ . Give each coefficient in its simplest form.

(6)

Use your answer to part (a) to find the binomial expansion in ascending powers of  $x$ , up to and including the term in  $x^3$ , of

(b)  $g(x) = \frac{6}{\sqrt[3]{9+4x}}, \quad |x| < \frac{9}{4},$

(1)

(c)  $h(x) = \frac{6}{\sqrt[3]{9-8x}}, \quad |x| < \frac{9}{8}.$

(2)

June 2012

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4. (a) Find the binomial expansion of

$$\sqrt[3]{(8-9x)}, \quad |x| < \frac{8}{9}$$

in ascending powers of  $x$ , up to and including the term in  $x^3$ . Give each coefficient as a simplified fraction.

(6)

- (b) Use your expansion to estimate an approximate value for  $\sqrt[3]{7100}$ , giving your answer to 4 decimal places. State the value of  $x$ , which you use in your expansion, and show all your working.

(3)

**June 2013 (R)**

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5. The curve  $C$  has equation

$$16y^3 + 9x^2y - 54x = 0.$$

- (a) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

(5)

- (b) Find the coordinates of the points on  $C$  where  $\frac{dy}{dx} = 0$ .

(7)

**June 2012**

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6. Water is being heated in a kettle. At time  $t$  seconds, the temperature of the water is  $\theta$  °C.

The rate of increase of the temperature of the water at any time  $t$  is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \quad \theta \leq 100$$

where  $\lambda$  is a positive constant.

Given that  $\theta = 20$  when  $t = 0$ ,

- (a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

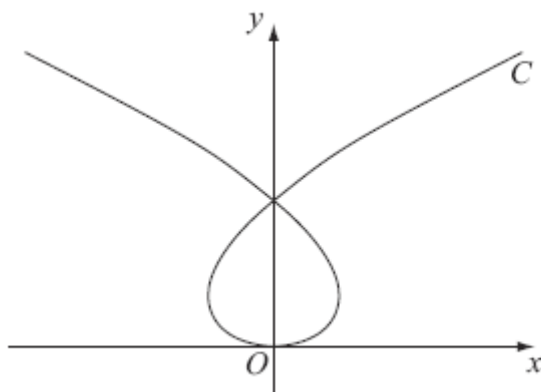
- (b) Given that  $\lambda = 0.01$ , find the time, to the nearest second, when the kettle switches off.

(3)

**June 2013**

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7.



**Figure 3**

The curve  $C$  shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where  $t$  is a parameter. Given that the point  $A$  has parameter  $t = -1$ ,

(a) find the coordinates of  $A$ .

**(1)**

The line  $l$  is the tangent to  $C$  at  $A$ .

(b) Show that an equation for  $l$  is  $2x - 5y - 9 = 0$ .

**(5)**

The line  $l$  also intersects the curve at the point  $B$ .

(c) Find the coordinates of  $B$ .

**(6)**

**January 2009**

8. With respect to a fixed origin  $O$ , the line  $l$  has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point  $A$  lies on  $l$  and has coordinates  $(3, -2, 6)$ .

The point  $P$  has position vector  $(-p\mathbf{i} + 2p\mathbf{k})$  relative to  $O$ , where  $p$  is a constant.

Given that vector  $\overrightarrow{PA}$  is perpendicular to  $l$ ,

(a) find the value of  $p$ .

(4)

Given also that  $B$  is a point on  $l$  such that  $\angle BPA = 45^\circ$ ,

(b) find the coordinates of the two possible positions of  $B$ .

(5)

June 2013

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**TOTAL FOR PAPER: 75 MARKS**

**END**

Question Number	Scheme	Marks
1.	$f(x) = \frac{1}{\sqrt{4+x}} = (4+x)^{-\frac{1}{2}}$ $= (4)^{-\frac{1}{2}} (1 + \dots)^{\dots} \qquad \frac{1}{2} (1 + \dots)^{\dots} \text{ or } \frac{1}{2\sqrt{1+\dots}}$ $= \dots \left( 1 + \left(-\frac{1}{2}\right) \left(\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(\frac{x}{4}\right)^2}{2} + \frac{\left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{x}{4}\right)^3}{3!} + \dots \right)$ $\qquad \qquad \qquad \text{ft their } \left(\frac{x}{4}\right)$ $= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \dots$	<p>M1</p> <p>B1</p> <p>M1 A1ft</p> <p>A1, A1 (6)</p> <p><b>(6 marks)</b></p>

Question Number	Scheme	Marks
2. (a)	<p>** represents a constant (which must be consistent for first accuracy mark)</p> <p><math>(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}</math></p> <p>Takes 8 outside the bracket to give any of <math>\underline{(8)^{\frac{1}{3}}}</math> or <math>\underline{2}</math>.</p> <p>Expands <math>(1+**x)^{\frac{1}{3}}</math> to give a simplified or an un-simplified <math>1+(\frac{1}{3})(**x)</math>;</p> <p><math>= 2\left\{1+(\frac{1}{3})(**x); +\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3 + \dots\right\}</math></p> <p>A correct simplified or an un-simplified <math>\{.....\}</math> expansion with candidate's followed through <math>(**x)</math></p> <p><b>with <math>** \neq 1</math></b></p>	<p><b>B1</b></p> <p>M1;</p> <p>A1 <math>\sqrt{}</math></p>
	<p><math>= 2\left\{1+(\frac{1}{3})(-\frac{3x}{8}) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\frac{3x}{8})^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-\frac{3x}{8})^3 + \dots\right\}</math></p> <p></p> <p><math>= 2\left\{1 - \frac{1}{8}x; -\frac{1}{64}x^2 - \frac{5}{1536}x^3 - \dots\right\}</math></p> <p><math>= 2 - \frac{1}{4}x; -\frac{1}{32}x^2 - \frac{5}{768}x^3 - \dots</math></p> <p>Either <math>2\{1 - \frac{1}{8}x \dots\}</math> or anything that cancels to <math>2 - \frac{1}{4}x</math>;</p> <p>Simplified <math>-\frac{1}{32}x^2 - \frac{5}{768}x^3</math></p>	<p>A1;</p> <p>A1</p> <p><b>[5]</b></p>
	<p>(b) <math>(7.7)^{\frac{1}{3}} \approx 2 - \frac{1}{4}(0.1) - \frac{1}{32}(0.1)^2 - \frac{5}{768}(0.1)^3 - \dots</math></p> <p><math>= 2 - 0.025 - 0.0003125 - 0.0000065104166\dots</math></p> <p><math>= 1.97468099\dots</math></p> <p><i>Attempt to substitute <math>x = 0.1</math> into a candidate's binomial expansion.</i></p> <p>awrt 1.9746810</p>	<p>M1</p> <p>A1</p> <p><b>[2]</b></p>
		<b>7 marks</b>

Question Number	Scheme	Marks
3. (a)	$f(x) = \dots (\dots - \dots x)^{-\frac{1}{2}}$ $= 6 \times 9^{-\frac{1}{2}} (\dots)$ $\frac{6}{9^{\frac{1}{2}}}, \frac{6}{3}, 2 \text{ or equivalent}$ $= \dots \left( 1 + \left(-\frac{1}{2}\right)(kx) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(kx)^3 + \dots \right)$ $= 2 \left( 1 + \frac{2}{9}x + \dots \right) \quad \text{or } 2 + \frac{4}{9}x$ $= 2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3 + \dots$	M1 B1 M1; A1ft A1 A1 <b>(6)</b>
(b)	$g(x) = 2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3 + \dots$	B1ft <b>(1)</b>
(c)	$h(x) = 2 + \frac{4}{9}(2x) + \frac{4}{27}(2x)^2 + \frac{40}{729}(2x)^3 + \dots$ $\left( = 2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3 + \dots \right)$	M1 A1 <b>(2)</b> <b>[9]</b>



Qn	Scheme	Mark
4. (a)	$\{\sqrt[3]{(8-9x)}\} = (8-9x)^{\frac{1}{3}}$ $= \underline{(8)^{\frac{1}{3}}}\left(1 - \frac{9x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1 - \frac{9x}{8}\right)^{\frac{1}{3}}$ $= \{2\} \left[ 1 + \left(\frac{1}{3}\right)(kx) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3 + \dots \right]$ $= \{2\} \left[ 1 + \left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x}{8}\right)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3 + \dots \right]$ $= 2 \left[ 1 - \frac{3}{8}x; -\frac{9}{64}x^2 - \frac{45}{512}x^3 + \dots \right]$ $= 2 - \frac{3}{4}x; -\frac{9}{32}x^2 - \frac{45}{256}x^3 + \dots$	Power of $\frac{1}{3}$ M1 $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ B1 M1 A1 A1; A1 <b>(6)</b>
	<b>(b)</b> $\{\sqrt[3]{7100} = 10\sqrt[3]{71} = 10\sqrt[3]{(8-9x)},\}$ so $x = 0.1$ When $x = 0.1$ , $\sqrt[3]{(8-9x)} \approx 2 - \frac{3}{4}(0.1) - \frac{9}{32}(0.1)^2 - \frac{45}{256}(0.1)^3 + \dots$ $= 2 - 0.075 - 0.0028125 - 0.00017578125$ $= 1.922011719$ So, $\sqrt[3]{7100} = 19.220117919\dots = \underline{19.2201}$ (4 dp)	Writes down or uses $x = 0.1$ B1 M1 19.2201 <b>cso</b> A1 <b>cao</b> <b>(3)</b> <b>[9]</b>

Question Number	Scheme	Marks
5. (a)	<p>Differentiating implicitly to obtain <math>\pm ay^2 \frac{dy}{dx}</math> and/or <math>\pm bx^2 \frac{dy}{dx}</math></p> <p><math>48y^2 \frac{dy}{dx} + \dots - 54 \dots</math></p> <p><math>9x^2 y \rightarrow 9x^2 \frac{dy}{dx} + 18xy</math> or equivalent</p> <p><math>(48y^2 + 9x^2) \frac{dy}{dx} + 18xy - 54 = 0</math></p> <p><math>\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2} \left( = \frac{18 - 6xy}{16y^2 + 3x^2} \right)</math></p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1 (5)</p>
(b)	<p><math>18 - 6xy = 0</math></p> <p>Using <math>x = \frac{3}{y}</math> or <math>y = \frac{3}{x}</math></p> <p><math>16y^3 + 9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right) = 0</math> or <math>16\left(\frac{3}{x}\right)^3 + 9x^2\left(\frac{3}{x}\right) - 54x = 0</math></p> <p>Leading to</p> <p><math>16y^4 + 81 - 162 = 0</math> or <math>16 + x^4 - 2x^4 = 0</math></p> <p><math>y^4 = \frac{81}{16}</math> or <math>x^4 = 16</math></p> <p><math>y = \frac{3}{2}, -\frac{3}{2}</math> or <math>x = 2, -2</math></p> <p>Substituting either of their values into <math>xy = 3</math> to obtain a value of the other variable.</p> <p><math>\left(2, \frac{3}{2}\right), \left(-2, -\frac{3}{2}\right)</math> both</p>	<p>M1</p> <p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1 (7)</p> <p>[12]</p>

Question Number	Scheme	Marks
6.	$\frac{d\theta}{dt} = \lambda(120 - \theta), \quad \theta \approx 100$	
(a)	$\int \frac{1}{120 - \theta} d\theta = \int \lambda dt \quad \text{or} \quad \int \frac{1}{\lambda(120 - \theta)} d\theta = \int dt$ $-\ln(120 - \theta); = \lambda t + c \quad \text{or} \quad -\frac{1}{\lambda} \ln(120 - \theta); = t + c$ $\{t = 0, \theta = 20 \Rightarrow\} -\ln(120 - 20) = \lambda(0) + c$ $c = -\ln 100 \Rightarrow -\ln(120 - \theta) = \lambda t - \ln 100$ <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p><b>then either...</b></p> <math display="block">-\lambda t = \ln(120 - \theta) - \ln 100</math> <math display="block">-\lambda t = \ln\left(\frac{120 - \theta}{100}\right)</math> <math display="block">e^{-\lambda t} = \frac{120 - \theta}{100}</math> <math display="block">100e^{-\lambda t} = 120 - \theta</math> </div> <div style="width: 45%;"> <p><b>or...</b></p> <math display="block">\lambda t = \ln 100 - \ln(120 - \theta)</math> <math display="block">\lambda t = \ln\left(\frac{100}{120 - \theta}\right)</math> <math display="block">e^{\lambda t} = \frac{100}{120 - \theta}</math> <math display="block">(120 - \theta)e^{\lambda t} = 100</math> <math display="block">\Rightarrow 120 - \theta = 100e^{-\lambda t}</math> </div> </div> <p style="text-align: center;">leading to <math>\theta = 120 - 100e^{-\lambda t}</math></p>	<p>B1</p> <p>M1 A1; M1 A1 M1</p> <p>dddM1</p> <p>A1 *</p>
(b)	$\{\lambda = 0.01, \theta = 100 \Rightarrow\} \quad 100 = 120 - 100e^{-0.01t}$ $\Rightarrow 100e^{-0.01t} = 120 - 100 \Rightarrow -0.01t = \ln\left(\frac{120 - 100}{100}\right)$ $t = \frac{1}{-0.01} \ln\left(\frac{120 - 100}{100}\right)$ $\left\{t = \frac{1}{-0.01} \ln\left(\frac{1}{5}\right) = 100 \ln 5\right\}$ $t = 160.94379... = 161 \text{ (s) (nearest second)}$	<p>M1</p> <p>dM1</p> <p>A1</p> <p>Uses correct order of operations by moving from <math>100 = 120 - 100e^{-0.01t}</math> to give <math>t = \dots</math> and <math>t = A \ln B</math>, where <math>B &gt; 0</math></p> <p>awrt 161</p>
		<p>(8)</p> <p>(3)</p> <p>[11]</p>

Question Number	Scheme	Marks
7	<p>(a) At <math>A</math>, <math>x = -1 + 8 = 7</math> &amp; <math>y = (-1)^2 = 1 \Rightarrow A(7,1)</math> <span style="float: right;"><math>A(7,1)</math></span></p>	B1
	<p>(b) <math>x = t^3 - 8t</math>, <math>y = t^2</math>,</p> $\frac{dx}{dt} = 3t^2 - 8, \quad \frac{dy}{dt} = 2t$ $\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$ <p style="text-align: right;">Their <math>\frac{dy}{dx}</math> divided by their <math>\frac{dx}{dt}</math> <span style="float: right;">M1</span> Correct <math>\frac{dy}{dx}</math> <span style="float: right;">A1</span></p> <p>At <math>A</math>, <math>m(T) = \frac{2(-1)}{3(-1)^2 - 8} = \frac{-2}{3-8} = \frac{-2}{-5} = \frac{2}{5}</math> <span style="float: right;">Substitutes for <math>t</math> to give any of the four underlined oe: A1</span></p> <p>T: <math>y - (\text{their } 1) = m_T(x - (\text{their } 7))</math> <span style="float: right;">Finding an equation of a tangent with their point and their tangent gradient or finds <math>c</math> and uses <math>y = (\text{their gradient})x + "c"</math>. dM1</span></p> <p>or <math>1 = \frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}</math></p> <p>Hence T: <math>y = \frac{2}{5}x - \frac{9}{5}</math></p> <p>gives T: <u><math>2x - 5y - 9 = 0</math></u> <span style="float: right;">AG <u><math>2x - 5y - 9 = 0</math></u> A1</span> <span style="float: right;">cso (5)</span></p> <p>(c) <math>2(t^3 - 8t) - 5t^2 - 9 = 0</math> <span style="float: right;">Substitution of both <math>x = t^3 - 8t</math> and <math>y = t^2</math> into T M1</span></p> $2t^3 - 5t^2 - 16t - 9 = 0$ <p><math>(t+1)\{(2t^2 - 7t - 9) = 0\}</math> <span style="float: right;">A realisation that <math>(t+1)</math> is a factor. dM1</span></p> <p><math>(t+1)\{(t+1)(2t-9) = 0\}</math></p> <p><math>\{t = -1 \text{ (at } A)\} \quad t = \frac{9}{2} \text{ at } B</math> <span style="float: right;"><math>t = \frac{9}{2}</math> A1</span></p> <p><math>x = \left(\frac{9}{2}\right)^2 - 8\left(\frac{9}{2}\right) = \frac{729}{8} - 36 = \frac{441}{8} = 55.125</math> or awrt 55.1 <span style="float: right;">Candidate uses their value of <math>t</math> to find either the <math>x</math> or <math>y</math> coordinate ddM1</span></p> <p><math>y = \left(\frac{9}{2}\right)^2 = \frac{81}{4} = 20.25</math> or awrt 20.3 <span style="float: right;">One of either <math>x</math> or <math>y</math> correct. A1 Both <math>x</math> and <math>y</math> correct. A1</span></p> <p>Hence <math>B\left(\frac{441}{8}, \frac{81}{4}\right)</math> <span style="float: right;">awrt (6)</span></p>	(1)
		[12]

Question Number	Scheme	Marks
8.	$l: \mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \quad A(3, -2, 6), \quad \overrightarrow{OP} = \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix}$	
(a)	$\left\{ \overrightarrow{PA} \right\} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix} \quad \left\{ \overrightarrow{AP} \right\} = \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$ $= \begin{pmatrix} 3+p \\ -2 \\ 6-2p \end{pmatrix} \quad = \begin{pmatrix} -3-p \\ 2 \\ 2p-6 \end{pmatrix}$ $\begin{pmatrix} 3+p \\ -2 \\ 6-2p \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = 6 + 2p - 4 - 6 + 2p = 0$ $p = 1$	<p>Finds the difference between <math>\overrightarrow{OA}</math> and <math>\overrightarrow{OP}</math>. Ignore labelling.</p> <p>Correct difference.</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 <b>cso</b></p>
(b)	$ AP  = \sqrt{4^2 + (-2)^2 + 4^2} \quad \text{or} \quad  AP  = \sqrt{(-4)^2 + 2^2 + (-4)^2}$ <p>So, <math>PA</math> or <math>AP = \sqrt{36}</math> or 6 <b>cao</b></p> <p>It follows that, <math>AB = "6" \{ = PA \}</math> or</p> $PB = "6\sqrt{2}" \{ = \sqrt{2} PA \}$ <p>{Note that <math>AB = "6" = 2(\text{the modulus of the direction vector of } l)</math>}</p> $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \pm 2 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \quad \text{or}$ $\overrightarrow{OB} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - 3 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \quad \text{and}$ $\overrightarrow{OB} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - 7 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ $= \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 \\ -6 \\ 8 \end{pmatrix}$	<p>M1</p> <p>A1 <b>cao</b></p> <p>B1 ft</p> <p>Uses a correct method in order to find both possible sets of coordinates of <math>B</math>.</p> <p>Both coordinates are correct.</p> <p>M1</p> <p>A1 <b>cao</b></p>
		<p>(4)</p> <p>(5)</p> <p>[9]</p>

### Question 1

This proved a suitable starting question and the majority of candidates gained 5 or 6 of the available 6 marks. Nearly all could obtain the index as  $-\frac{1}{2}$  but there were a minority of candidates who had difficulty in factorising out 4 from the brackets and obtaining the correct multiplying constant of  $\frac{1}{2}$ . Candidates' knowledge of the binomial expansion itself was good and, even if they had an incorrect index, they could gain the method mark here. An unexpected number of candidates seemed to lose the thread of the question and, having earlier obtained the correct multiplying factor  $\frac{1}{2}$  and expanded  $\left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$  correctly, forgot to multiply their expansion by  $\frac{1}{2}$ .

### Question 2

In part (a), a majority of candidates produced correct solutions, but a minority of candidates were unable to carry out the first step of writing  $(8-3x)^{\frac{1}{3}}$  as  $2\left(1 - \frac{3x}{8}\right)^{\frac{1}{3}}$ . Those who did so were able to complete the remainder of this part but some bracketing errors, sign errors and manipulation errors were seen.

In part (b), many candidates realised that they were required to substitute  $x=0.1$  into their binomial expansion. About half of the candidates were able to offer the correct answer to 7 decimal places, but some candidates made calculation errors even after finding the correct binomial expansion in part (a). A few candidates used their calculator to evaluate the cube root of 7.7 and received no credit.

### Question 3

A small number of candidates used the binomial expansion with index  $\frac{1}{2}$  but the great majority used the correct index,  $-\frac{1}{2}$ , and were able to expand an expression of the form  $(1-kx)^{-\frac{1}{2}}$  correctly to obtain at least three marks. Although many dealt with the 9 correctly, taking  $9^{-\frac{1}{2}}$  outside a bracket, some did not combine it correctly with the 6, multiplying their binomial by 18 rather than 2. Full marks were common in part (a). In part (b), most realised that a change in signs was necessary but many changed the sign of the term in  $x^2$  as well as the terms in  $x$  and  $x^3$ . Part (c) was less well done than part (b) and many multiplied all three of the terms in  $x$ ,  $x^2$  and  $x^3$  by 2 instead of by 2, 4 and 8 respectively. Questions like parts (b) and (c) have rarely been set on these papers and it was clear that many candidates were not able to think their way into a solution that did not require a practised technique.

### Question 4

This question was generally well answered with about 76% of candidates gaining at least 6 of the 9 marks available and about 17% of candidates gaining all 9 marks. Part (a) was accessible with most candidates scoring all 6 marks and part (b) was discriminating and challenged the more able candidates.

In part (a), most candidates manipulated  $\sqrt[3]{(8-9x)}$  to give  $2\left(1-\frac{9x}{8}\right)^{\frac{1}{3}}$ , with the 2 outside the brackets sometimes written incorrectly as either 1 or  $\frac{1}{2}$  and a few incorrectly used a power of  $\frac{3}{2}$ . Many candidates were able to use a correct method for expanding a binomial expression of the form  $(1+ax)^n$ . A variety of incorrect values of  $a$  were seen, with the most common being  $\frac{9}{8}$ . Some candidates, having correctly expanded  $\left(1-\frac{9x}{8}\right)^{\frac{1}{3}}$ , forgot to multiply their expansion by 2. Sign errors, bracketing errors, and simplification errors were also seen in this part.

In part (b), the majority of candidates solved the equation  $\sqrt[3]{(8-9x)} = \sqrt[3]{1700}$  to give an answer of  $x = -788$ . These candidates substituted this value of  $x$  into the answer they had found in part (a), even though the question states that the binomial expansion is only valid for  $|x| < \frac{8}{9}$ . Only a minority of candidates realised that they needed to simplify  $\sqrt[3]{1700}$  to  $10\sqrt[3]{7.1}$  before deducing that they needed to substitute  $x = 0.1$  into their binomial expansion. Most of these candidates achieved the correct approximation of 19.2201, although a few forgot to multiply by 10 at the end and wrote 1.9220.

## Question 5

As has been noticed more than once in recent years, the topic of implicit differentiation is well understood and full marks in part (a) were very common. Mistakes mainly concerned the differentiation of  $9x^2y$ , involving a misinterpretation of the product rule.

Part (b) proved a test even for the most able. Most recognised that the numerator of their answer to (a) had to be equated to zero and obtained  $xy = 3$  or an equivalent but then many just gave up then immediately. It was disappointing to see a significant minority of those who realised that they should solve the simultaneous equations  $xy = 3$  and  $16y^3 + 9x^2y - 54x = 0$ , started by transforming  $xy = 3$  to  $y = 3x$ . Those who did start correctly often had problems with the resulting algebra and had difficulty reaching the correct  $x^4 = 16$  or  $y^4 = \frac{81}{16}$ .

Those who got this far often failed to realise that these equations have two solutions. Those who had correct values for either  $x$  or  $y$  could complete quickly by substituting into  $xy = 3$  but some made extra work for themselves by either starting all over again and finding the other variable independently or by substituting into  $16y^3 + 9x^2y - 54x = 0$ . The latter was particularly unfortunate if  $x$  had been found first as this resulted in a cubic in  $y$  which is difficult to solve. This question was a very discriminating one and it may be worth noting that the proportion of those who gained full marks on this question was slightly less than the proportion of those gaining the equivalent of a grade A on this paper.

## Question 6

This question was generally well answered with about 57% of candidates gaining at least 8 of the 11 marks available and about 37% of candidates gaining all 7 marks. A minority of candidates made no creditable attempt in part (a) and then scored full marks in part (b).

In part (a), those candidates who were able to separate the variables, were usually able to integrate both sides correctly, although a number integrated  $\frac{1}{120 - \theta}$  incorrectly to give

$\ln(120 - \theta)$ . Many candidates substituted  $t = 0, \theta = 120$  immediately after integration, to find their constant of integration as  $-\ln 100$  and most used a variety of correct methods to eliminate logarithms in order to achieve the printed result. A significant number of candidates, however, correctly rearranged their integrated expression into the form  $120 - \theta = Ae^{-\lambda t}$ , before using  $t = 0, \theta = 120$  to correctly find  $A$ . Common errors in this part included omitting the constant of integration, treating  $\lambda$  as a variable and incorrect manipulation in order to fudge the printed result. Also, a number of candidates struggled to remove logarithms correctly and gave an equation of the form  $120 - \theta = e^{-\lambda t} + e^c$  which was then sometimes manipulated to  $120 - \theta = Ae^{-\lambda t}$ .

In part (b), most candidates were able to substitute the given values into the printed equation and achieve  $t = 161$  seconds. Some candidates made careless errors when manipulating their expressions, whilst a number did not round their answer of 160.94... to the nearest second. Few candidates substituted the given values into their incorrect answer from part (a).

### Question 7

Part (a) was answered correctly by almost all candidates. In part (b), many candidates correctly applied the method of finding a tangent by using parametric differentiation to give the answer in the correct form. Few candidates tried to eliminate  $t$  to find a Cartesian equation for  $C$ , but these candidates were usually not able to find the correct gradient at  $A$ .

In part (c), fully correct solutions were much less frequently seen. A significant number of candidates were able to obtain an equation in one variable to score the first method mark, but were then unsure about how to proceed. Successful candidates mostly formed an equation in  $t$ , used the fact that  $t + 1$  was a factor and applied the factor theorem in order for them to find  $t$  at the point  $B$ . They then substituted this  $t$  into the parametric equations to find the coordinates of  $B$ . Those candidates who initially formed an equation in  $y$  only went no further. A common misconception in part (c), was for candidates to believe that the gradient at the point  $B$  would be the same as the gradient at the point  $A$  and a significant minority of candidates attempted to solve  $\frac{2t}{3t^2 - 8} = \frac{2}{5}$  to find  $t$  at the point  $B$ .

### Question 8

In general, this was the most poorly answered question on the paper with about 25% of candidates failing to score. Some candidates did not seem to have a firm grasp of what was required and many produced pages of irrelevant working. This question did discriminate well between candidates of average to higher abilities, with about 54% of candidates gaining at least 4 of the 9 marks available and only about 10% of candidates gaining all 9 marks. Part (a) was found to be fairly accessible, and part (b) was challenging to all but the most able candidates. Many were unable to think about the question logically or produce a clear diagram and establish the relationship between the length of  $AB$  (and/or  $PB$ ) and the length of  $PA$ .

In part (a), the vector  $\overrightarrow{PA}$  (or  $\overrightarrow{AP}$ ) was usually found, although sign slips, adding  $\overrightarrow{OP}$  to  $\overrightarrow{OA}$  and mixing up of the  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  components of  $\overrightarrow{OP}$  were common errors. Many candidates found the correct value of  $p$  by applying the correct scalar product between  $\overrightarrow{PA}$  (or  $\overrightarrow{AP}$ ) and



the direction vector  $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  and setting the result equal to 0, although some candidates used  $\overrightarrow{OA}$ ,  $2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  or  $13\mathbf{i} + 8\mathbf{j} + \mathbf{k}$  instead of  $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ . Other errors included taking the dot product between  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$  or deducing  $p = -1$  from a correct  $4p - 4 = 0$ .

Those candidates who attempted part (b) usually managed to find the magnitude of  $PA$  and many drew a diagram of triangle  $PAB$  correctly and deduced  $PA = AB$ . From this point, however, many candidates did not know how to proceed further, resulting in a lot of incorrect work which yielded no further marks. Some candidates, however, were able to form a correct equation in order to find both values of  $\lambda$ . It was unfortunate that a few, having found the

correct values of  $\lambda = -3$  and  $\lambda = -7$  then substituted these into  $\begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$  instead of the

equation for the line  $l$ . The most popular method for finding correct values of  $\lambda$  was for candidates to form and solve a Pythagorean equation in  $\lambda$  of  $AB = 6$  or  $AB^2 = 36$ . Other successful methods for finding  $\lambda$  included solving  $PB = 6\sqrt{2}$  or solving a dot product equation between either  $\overrightarrow{PA}$  and  $\overrightarrow{PB}$  or  $\overrightarrow{AB}$  and  $\overrightarrow{PB}$ .

Few candidates realised that the length  $AB$  was twice the length of the direction vector of the line  $l$  and applied twice the direction vector  $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  in either direction from  $A$  in order to find both positions for  $B$ .

## Statistics for C4 Practice Paper Silver Level S1

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	6		83	4.96		5.59	5.19	4.77	4.16	3.41	2.12
2	7		73	5.12		6.21	5.13	4.28	3.51	2.96	1.65
3	9		74	6.65	8.56	7.64	6.85	6.06	5.05	3.92	2.32
4	9		66	5.97	7.61	6.28	5.73	4.45	4.18	3.11	1.32
5	12		63	7.60	11.50	9.41	7.62	5.96	4.53	3.16	1.69
6	11	11	64	7.04	10.81	9.8	7.67	5.12	3.26	1.97	0.88
7	12		60	7.25		8.99	6.53	5.57	4.20	3.05	1.29
8	9	0	41	3.71	7.65	5.45	3.63	1.93	0.97	0.53	0.21
	<b>75</b>		<b>64</b>	<b>48.30</b>		<b>59.37</b>	<b>48.35</b>	<b>38.14</b>	<b>29.86</b>	<b>22.11</b>	<b>11.48</b>